

Chapter 1 – Exercises

Exercise 1 (*ODEs vocabulary*)

For each ODE below, determine: (i) its order; (ii) whether it is linear or nonlinear; (iii) whether it is homogeneous; (iv) whether it has constant coefficients; (v) whether it is autonomous. For the autonomous equations, also find all equilibrium solutions.

1. $y'(t) + 2y(t) = 0$

4. $y'(t) + y(t) = \cos(t)$

2. $y''(t) - y'(t) - 6y(t) = 0$

5. $y'(t) = y(t)(3 - y(t))$

3. $y'(t) = t^2y(t)$

6. $y''(t) + 3ty'(t) - y(t) = 5$

Exercise 2 (*Solving homogeneous ODEs*)

Solve the following homogeneous ODEs:

1. $y'(t) + ty(t) = 0$

5. $\begin{cases} y'(t) + ty(t) = 0 \\ y(1) = 5 \end{cases}$

2. $2y'(t) + \sin(t)y(t) = 0$

6. $\begin{cases} (1 + t^2)y'(t) + ty(t) = 0 \\ y(0) = 1 \end{cases}$

3. $y'(t) + \sqrt{t}y(t) = 0$

4. $(1 + t^2)y'(t) + y(t) = 0$

Exercise 3 (*Solving non-homogeneous ODEs*)

Solve the following non-homogeneous ODEs:

1. $y'(t) + 5y(t) = 6$

5. $y'(t) - y(t) = (t + 1)e^t$

2. $y'(t) - 4y(t) = e^{4t}$

6. $7y'(t) + 2y(t) = 2t^3 - 5t^2 + 4t - 1$

3. $y'(t) + 4y(t) = t^2$

7. $y'(t) + 3y(t) = (2t - 1)e^{-3t}$

4. $y'(t) + y(t) = 2\sin(t)$

8. $2y'(t) + 2y(t) = 4\cos(3t)$

Exercise 4 (*Slope fields and phase diagrams*)

For each of the following differential equations, sketch the direction field and draw several solution curves corresponding to different initial conditions. When possible, solve the equation analytically and compare the exact solution with your sketches.

1. $y'(t) = -y/t$.

2. $y'(t) = 2t + y$.

3. $y'(t) = t^2 + y^2 - 1$.